### Stage 2 - Addition and Subtraction

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<tr>
<td>A student:</td>
<td>› uses appropriate terminology to describe, and symbols to represent, mathematical ideas MA2-1WM</td>
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<td>› selects and uses appropriate mental or written strategies, or technology, to solve problems MA2-2WM</td>
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<td>› checks the accuracy of a statement and explains the reasoning used MA2-3WM</td>
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<td></td>
<td>› uses mental and written strategies for addition and subtraction involving two-, three-, four and five-digit numbers MA2-5NA</td>
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### Language
Students should be able to communicate using the following language: plus, add, **addition**, minus, the difference between, **subtract**, **subtraction**, equals, is equal to, is the same as, number sentence, empty number line, strategy, **digit**, **estimate**, **round to**.

Students need to understand the different uses for the = sign, eg 4 + 1 = 5, where the = sign indicates that the right side of the number sentence contains 'the answer' and should be read to mean 'equals', compared to a statement of equality such as 4 + 1 = 3 + 2, where the = sign should be read to mean 'is the same as'.

### Ignition Activities

**Greedy Pig**

1. To play this game you need an ordinary 6-sided die.
2. Each turn of the game consists of one or more rolls of the die. You keep rolling until you decide to stop, or until you roll a 1. You may choose to stop at any time.
3. If you roll a 1, your score for that turn is 0.
4. If you choose to stop rolling before you roll a 1, your score is the sum of all the numbers you rolled on that turn.
5. Each player has 10 turns.
6. The player with the highest score wins.

There are many variations of this game, the most common being a full class version in which the teacher rolls the die, and calls out the numbers. All students play using the same numbers and their score depends on when they elect to ‘save’ their score. If they save their score any further rolls that turn do not count towards their score. If a 1 is rolled all players who have not saved their score get 0 for that turn and the next turn starts. The ones dice can be changed to adding tens, hundreds or thousands by writing on blank dice. 1 could be changed to any other number as the key number to avoid rolling.
## Explicit Mathematical Teaching

Addition and Subtraction should move from counting and combining perceptual objects, to using numbers as replacements for completed counts with mental strategies, to recordings that support mental strategies (such as jump or split, partitioning or compensation).

Subtraction typically covers three different situations:
- ‘taking away’ from a group
- ‘comparing’ two groups
- finding ‘how many more’.

Students should be confident with the taking away from a group before being introduced to ‘comparing’ two groups. Students should be able to compare groups of objects before being asked to find out how many more or how many less there are in a group.

Modelling, drawing and writing mathematical problems should be encouraged.

Model using Newman’s Analysis steps for solving word problems. (Refer to attached sheet)

The word ‘left’ can be ambiguous eg ‘There were five children in the room. Three went to lunch. How many left?’ Is the question asking how many children are remaining in the room or how many children went to lunch?

Make sure a range of problems are used including adding and subtracting money, varying where the unknown quantity appears eg

- \[32 + 68 = ?\]
- \[? + 68 = 100\]
- \[32 + ? = 100\]

Ask ‘Where do we use addition or subtraction? Brainstorm language used on charts.

Provide explicit instruction. Start with addition and subtraction problems. Teach these together so that students learn to discriminate. Model how key phrases are translated to symbols.

Ask students to make up a word question to result in 23 – 5. Note what words were used to show the subtraction eg. 23 girls went swimming and five were eaten by sharks. 23 Ants were marching and five were exterminated by Mortein. Discuss where the language of Mathematics fits in to the language of everyday.

Model the jump, split, compensation, bridging, extending number facts, changing order of addends strategies.

- **the jump strategy**
  \[23 + 35; 23 + 30 = 53, 53 + 5 = 58\]

- **the split strategy**
  \[23 + 35; 20 + 30 + 3 + 5 = 58\]

- **the compensation strategy**
  \[63 + 29; 63 + 30 = 93, \text{subtract} 1, \text{to obtain} 92\]

- **using patterns to extend number facts**
  \[5 - 2 = 3, \text{so} 500 - 200 = 300\]

- **bridging the decades**
  \[34 + 17; 34 + 10 = 44, 44 + 7 = 51\]

- **changing the order of addends to form multiples of 10**
  \[16 + 8 + 4; \text{add} 16 \text{and} 4 \text{first}\]
- using place value to partition numbers, eg \(2500 + 670: 2500 + 600 + 70 = 3170\)
- partitioning numbers in non-standard forms, eg \(500 + 670: 670 = 500 + 170, so\)
\(500 + 670 = 500 + 500 + 170, which is 1000 + 170 = 1170\)
- recording mental strategies
  eg \(159 + 22: \) 'I added 20 to 159 to get 179, then I added 2 more to get 181.'
or, on an empty number line
  \(159\quad 169\quad 179\quad 180\quad 181\)
- adding and subtracting two or more numbers, with and without trading, using concrete materials and recording their method
- using a formal written algorithm and applying place value
to solve addition and subtraction problems, involving two, three- and four-digit numbers
  eg \(134 + 2459 + 568 - 1353 = \quad 253\quad 138\quad 322\quad 168\)
Work examples using a variety of number lines eg chalk ones out in playground, strips with pegs, empty number lines. Explain process that they used to get solution.
Discuss how subtraction is opposite to addition – 'inverse operations'
Discuss how some strategies work well for both and some don’t eg split strategy doesn’t work well for subtraction.
Students should be encouraged to estimate answers before attempting to solve problems in concrete or symbolic form. There is still a need to emphasise mental computation even though students can now use a formal written method. The following formal method may be used.
Use the equals sign to record equivalent number sentences involving addition and subtraction and so to mean 'is the same as', rather than to mean to perform an operation, eg \(32 - 13 = 30 - 11\)
- check given number sentences to determine if they are true or false and explain why, eg 'Is \(39 - 12 = 15 + 11\) true? Why or why not?'

Recognise and explain the connection between addition and subtraction (ACMNA054)
- demonstrate how addition and subtraction are inverse operations
- explain and check solutions to problems, including by using the inverse operation
Represent money values in multiple ways and count the change required for simple transactions to the nearest five cents *(ACMNA059)*

- calculate equivalent amounts of money using different denominations, eg 70 cents can be made up of three 20-cent coins and a 10-cent coin, or two 20-cent coins and three 10-cent coins, etc
- perform simple calculations with money, including finding change, and round to the nearest five cents
- calculate mentally to give change

**Whole Class Teaching Activities**

Using the 2008 NAPLAN Year 3 question 15(Jim is 91 years old. Sam is 8 years old. What is the difference in their ages?), ask “What do I know?” Answer: Jim is 91 years old. Sam is 8 years old. What do I want to know? Answer: What is the difference in their ages?

Think aloud: “I know how old Jim is and I know Sam is 8. I know that difference is another word for take away. I will solve this problem using subtraction”

Model the solution of the problem using concrete materials and/or a drawing and writing the appropriate number sentence on the board.

**Step 1:** ‘How many to start with? Jim is 91.’ Put out 91 using tens and ones or draw this.

**Step 2:** ‘How old is Sam?’ Answer: 8. Move 8 ones away or cross out 8 drawn ones. Write $91 - 8 = $ How many left?’ Answer: 83. Write $91 - 8 = 83$

**Step 3:** ‘What is the difference in their ages?’ Answer: 83. Discuss and model other possible solution strategies with the group using counters or drawings.

Guide students to translate the information into a number sentence. 91 take away 8 means I subtract so I use the − sign. I want to know many are left so I use the equal sign =. My number sentence is $91 - 8 = 83$

Explain and check solutions to problems, including by using the inverse operation

**Mental Strategies**

Students are asked to calculate $34 + 17$ in their heads. They are then asked to record the strategy they used. This process is repeated for other problems, such as:

- $73 - 25 162 - 69$
- $63 + 29 188 - 89$

Students discuss which methods are the most efficient.

**Extension:** Students are given increasingly more difficult problems to solve mentally. Students explain and discuss the strategies they use eg for ‘$188 - 89 = ?$’ A student may say, ‘I took away 88 and that was easy because it left 100 but I had to take away one more, because $88 + 1 = 89$, so the answer is ‘99.’ Students record the mental strategies they use.

Possible questions include:

- is there a better strategy?
- what is the best method to find a solution to this problem?

Explain and check solutions to problems, including by using the inverse operation

**Recording on Empty Number Lines**

Students are shown the number sentence $157 + 22$ and an empty number line. The teacher marks the number 157 on the number line.
Possible questions include:
- what is the next multiple of ten after 157?
- how many do you add on to get that number?
Students record their answers on the number line.
Possible questions include:
- can you work it out with fewer steps?
- can you visualise the number line in your head and do it?
- can you write the numbers on paper to help you keep track?
Explain and check solutions to problems, including by using the inverse operation

### Differences on Number Lines
In pairs, students draw an empty number line. Student A chooses two three-digit numbers and places them on the number line. Student B uses the number line to work out and record the difference between the two numbers. Students explain the mental strategies they used to find the answer. They reflect on their method, considering whether it can be improved.
Explain and check solutions to problems, including by using the inverse operation

### Using Maths Tracks-Stage 2A, Unit 21-Number
One of a series of teaching units to accompany the Rigby/Harcourt series 'Maths Tracks'. Student activities include using mental strategies for addition and subtraction involving two, three-digit numbers and four-digit numbers, including using patterns to extend number facts; recording mental strategies. Meets BoS outcomes NS2.2, WMS2.2, WMS2.4. Click on link below.

### Base 10 Material
Students use 2, 3 or 4 dice to generate a two-, three- or four digit number and then represent this number using Base 10 material. Students then generate a second, smaller number by rolling one less die. Students represent this number using Base 10 material, then add the two numbers and show the result using Base 10 material. Students repeat this process, subtracting the second number from the first. Students record their solutions.

### Number Charts (1-1000)
Children learn to add and subtract ones and tens by moving left, right, up and down on charts with numbers up to 1000 eg 45 + 20 etc

### Linking 3
Students record sixteen different numbers between 1 and 50 in a 4 x 4 grid
eg

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<tbody>
<tr>
<td>19</td>
<td>28</td>
<td>17</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>41</td>
<td>5</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>38</td>
<td>49</td>
</tr>
<tr>
<td>15</td>
<td>26</td>
<td>40</td>
<td>7</td>
</tr>
</tbody>
</table>
Students link and add three numbers vertically or horizontally. Possible questions include:
- can you find links that have a total of more than 50?
- can you find links that have a total of less than 50?
- how many links can you find that have a total that is a multiple of 10?
- what is the smallest/largest total you can find?
- can you find ten even/odd totals?

**Estimating Addition of Three-Digit Numbers**
The teacher briefly displays the numbers 314, 311, 310, 316, 312 on cards, then turns the cards over so that the numbers be seen. Students are asked to estimate the total and give their reasons. The teacher reveals the numbers one at a time so that the students can find the total. The task could be repeated with other three-digit numbers and with four-digit numbers.

**Take-away Reversals**
In pairs, students choose a three-digit number without repeating any digit and without using zero eg 381. The student reverses the order of the digits to create a second number ie 183. The student subtracts the smaller number from the larger and records this as a number sentence. The answer is used to start another reversal subtraction. Play continues until zero is reached. The process could be repeated for other three or four digit numbers. Students discuss their work and any patterns they have observed.

Note: may not always get to zero.

**Estimating Differences**
The teacher shows a card with the subtraction of a pair of two-digit numbers eg 78 – 32. Students estimate whether the difference between the numbers is closer to 10, 20, 30, 40 or 50 and give reasons why. The teacher shows other cards eg 51 – 18, 60 – 29, 43 – 25, 33 – 25. Students estimate the differences and discuss their strategies. They are asked to think about rounding numbers on purpose. For example for 51 – 18, students may round 51 down to 50 and 18 up to 20.

**The Answer Is ...**
Students construct subtraction number sentences with the answer 123. Students are challenged to include number sentences involving four-digit numbers.

**How Much?**
Students are told that a sofa and a desk cost $1116. If the sofa costs $700 more than the desk, how much does the desk cost? Students discuss. Students could pose other similar problems to solve such as 'What does each item cost if together they cost $1054 and one was $643 more than the other?'

Possible questions include:
- what strategy did you choose to use and why?
- what was the key word/s in understanding the problem?
- how could you check that you have the correct solution?
• could there be more than one solution?
• calculate equivalent amounts of money using different denominations, eg 70 cents can be made up of three 20-cent coins and a 10-cent coin, or two 20-cent coins and three 10-cent coins, etc
• perform simple calculations with money, including finding change, and round to the nearest five cents
  • calculate mentally to give change

### Inverse operations

Students need to opportunities to demonstrate how addition and subtraction are inverse operations. This video shows a series of three different clips of real life situations are shown in a quiz situation to help the participants work out number facts related to adding and subtracting. They work out the matching number facts which illustrate that subtraction is the inverse of addition. Adding and then subtracting the same amount gets you back to where you started.

http://www.bbc.co.uk/learningzone/clips/addition-and-subtraction-inverse-operations/3134.html

To understand that subtraction is the inverse to addition

Choose 3 of the given number cards (numbers in the range of 10-20) to write an addition number sentence and the corresponding subtraction number sentence.

For further resources please visit: http://www.primaryresources.co.uk/maths/mathsC1b.htm

### Guided Group and Independent Activities

Please look at Developing Efficient Numeracy Strategies 1 and 2 (blue and orange ones) and Sample Units of Work for more ideas on how to cater for the needs of your different children in your Maths groups

### Totalling Up

Hand out containers containing different amounts of money in 5c, 10c, 20c, 50c, $1, $2 coins. Imagine it is money collected from drink sales at a school disco and needs counting. In small groups ask them to count each amount, deciding which way would be best. Deliberately give different assortments of coins to different groups. Ask each group to sort the coins out and total up each amount separately before finding the overall amount; keeping a record as they go. Ask them to think about quick ways of adding it up. Model their methods on the board:

• calculate equivalent amounts of money using different denominations, eg 70 cents can be made up of three 20-cent coins and a 10-cent coin, or two 20-cent coins and three 10-cent coins, etc
• perform simple calculations with money, including finding change, and round to the nearest five cents
• calculate mentally to give change

Explain and check solutions to problems, including by using the inverse operation
## Four Turns To 100

Organise the students into groups of four. Provide each group of students with a pack of cards in the range 1 to 9. Each player draws a card from the deck and decides if the number they have drawn will represent ones or tens. For example, if a five is drawn it can represent five or fifty. The players take a second draw from the pack, again nominating if the number represents tens or ones and adds the number to their first card. Have the students record their total on an empty number line. Continue the activity until each student has drawn four cards. The player with the highest total not exceeding 100 wins.

**Variations**

Players start at 100 and subtract the numbers, after nominating if the number drawn represents tens or ones. The player closest to zero is the winner. Players draw two numbers from the pile and make the highest two digit number possible. This becomes their starting number and they continue to play as in the above variation.

## Teeny Tiny Ten-Frames

Provide the students with a set of *Teeny Tiny ten-frames*. Nominate a two digit number and ask the students to represent the number using the ten-frames. Have the students share how they made the number. Ask the students to make a second two-digit number. Repeat the questioning.

Have the students find the total of the two numbers using the ten-frames. Discuss how they solved the addition.

**Variation**

Make the first number and then cover it up. Make the second number and use the material to determine the total of both numbers.

## Number Cards

Students make number cards from 1 to 9 as shown.

```
1 2 3 4 5 6 7 8 9
```

Students use these cards to make two three-digit numbers that add to give the largest total possible and the smallest total possible eg Given 4, 5, 2 and 3, 1, 6:

- Largest total possible is 542 + 631 = 1173
- Smallest total possible is 245 + 136 = 381

Students arrange the cards to make three three-digit numbers that add up to 999. Students are challenged to find as many solutions as they can.

Explain and check solutions to problems, including by using the inverse operation.

## Cross-over

In pairs, students each choose a number between 1 and 1000. The student with the larger number always subtracts a number from their chosen number. The student with the smaller number always adds a number to their chosen number. The student who is adding must always have a number less than their partner’s answer. The student who is subtracting must always have a number more than their partner’s answer. Play continues until one student is forced to ‘cross over’ their partner’s number. The student who crosses over their partner’s number loses the game.
Tracks
Organise the students into pairs and provide them with a copy of Tracks BLM, a set of numeral cards 0–9 and a hundred chart. Have the students take turns to draw two cards from the deck to make a two-digit number. The student who has drawn the cards records this number on the “Tracks” sheet as their starting number. The partner then fills in the boxes on the sheet with three directional arrows. These arrows indicate if the student is to:

- ![1] count back by ten from the number
- ![2] count on by ten
- ![3] add on one
- ![4] take away one.

The first student locates the starting number on the hundred chart and follows the directional arrows to determine the number they would finish on.

For example, if the starting number is 24 and the directional arrows were, ![1], ![3], ![4], then the finishing number would be 45.

Developing Efficient Numeracy Strategies 2 (DENS 2) pg 86-87
Tracks Blackline Master pg 154

Highway Racer
Have the students work in pairs so that each student can explain and verify calculations. Prepare Highway Racer.
racer worksheets for each pair of students. To complete the worksheet, the students take turns to mentally calculate, and record, the number needed to be added or subtracted in order to move to the total written in the next box.

**Variations**
- Have the students create their own “race tracks” for others to solve.
- Have the students verify their partner’s answers using a calculator.
- Have one of the players time his or her partner from “start” to “finish” and then swap roles.
- Have the students “race the clock”. For example, *How far can you move along the track in 60 seconds?*

**Problem Solving**
Kim’s meal at a restaurant cost half as much as her dad’s meal. Kim and her dad paid $18 altogether for their friends.

**Engineer’s Dice**
Provide each group of students with five dice. To play the game a target number is selected by the group. The students then take turns to roll the dice in the following way:

- Roll all five dice. Choose two of the dice and nominate an operation (+ - ÷) to carry out with the numbers rolled. Record the result. Discard these two dice.
- Roll the remaining three dice. Choose one number rolled, complete another operation (+ - ÷) with the chosen number and the first score. Discard that die.
- Roll the remaining two dice. Choose one number rolled and complete the same process as the step above using the current total.
- Roll the last die and complete the same process using the current total.

After each player has had his or her turn, the students compare their totals to see who is closest to the target score.

**Investigation:**
- How many different ways can you add 57 + 35 in your head? Write number sentences to explain your methods.
- Explain and check solutions to problems, including by using the inverse operation

**Trading Games**
The trading games Win 500 or Lose 500 can be adapted for Stage 2 by adding and subtracting two-digit...
numbers using, recording and evaluating mental strategies. Students are given a pack of playing cards with the tens and the picture cards removed. The Aces are retained and represent 1 and the Jokers are retained and represent 0. Students flip two cards and assign place values to the numbers turned over. Students play Win 5000/50 000 and Lose 5000/50 000 to add and subtract three-digit and four-digit numbers. Students estimate their answer and then use formal written algorithms. Students could use a calculator to check their answer. Students are encouraged to pose problems, including money problems, using their numbers.

Explain and check solutions to problems, including by using the inverse operation

Recording equivalent number sentences
Students are given a pack of cards with digits 0-9 and operation cards of - + =. Students create 2 two, three or four digit numbers. They then select a operations card and perform the operation. Students then need to record equivalent number sentences ensuring the = is used to show this.
use the equals sign to record equivalent number sentences involving addition and subtraction and so to mean 'is the same as', rather than to mean to perform an operation, eg 32 – 13 = 30 – 11
check given number sentences to determine if they are true or false and explain why, eg 'Is 39 – 12 = 15 + 11 true? Why or why not?
Variation: Students can record their original number sentence and swap with other groups for them to complete.

Computer Learning Objects

4 Turns to 100-Stages 1-2

Addition Wheel –Stages 1-2

Wishball –Stages 2-3

~ 11 ~
Using Learning Objects To Teach Mathematics' CD ROM
Or
Count Me In Too website
(Click on link below)

Teaching and Learning Exchange
www.tale.edu.au
Type in reference number into search box, click on link and then click view
(download it if you want to use it a lot)

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<td>Pre Assessment</td>
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<tr>
<td>Children work out addition/subtraction equations using as many different strategies as they can eg. Jump, split, compensation, bridging, extending number facts, changing order of addends ((13 + 15 + 7 = 13 + 7 + 15))</td>
</tr>
</tbody>
</table>

| Count Me In Too : SENA 1, SENA 2 |
| Select activities to identify where students are on the CMIT framework in number for Addition and subtraction |

| What is the question? Write three difficult questions where 65 is the answer. Draw a circle around what you think is the hardest question for a student to answer |

| Children solve a variety of two-, three- and four-digit addition and subtraction equations, using as many strategies as they can. Students explain methods. |

| Two and three digit addition and subtraction |
| Provide the students with a blank piece of paper and ask them to fold the paper into quarters. Write on the board two addition and two subtraction problems, eg 78 + 36, 345 + 189, 95 – 46 and 800 – 241. Ask the students to solve each problem, using a quarter of the paper, recording the strategy they used. |

| Work it out in your head |
| Download the assessment proforma. |

<p>| Strategies for addition and subtraction |
| Explain 3 different ways to solve 257 + ? = 735. |
| Show how you would use an empty number line to solve 63 – 27. |</p>
<table>
<thead>
<tr>
<th>103 – 47 = 144 Explain where you think this student made errors.</th>
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<tbody>
<tr>
<td><strong>Helping Lee with subtraction</strong></td>
</tr>
<tr>
<td>Download the assessment proforma.</td>
</tr>
<tr>
<td><strong>Adding and subtraction with 2 and 3 digit numbers</strong></td>
</tr>
<tr>
<td>Students fold a piece of paper into 4 sections and write two addition and two subtraction problems provided by the teacher. Students explain how they solved each problem. Download the assessment proforma.</td>
</tr>
<tr>
<td><strong>Sheep and ducks</strong></td>
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<tr>
<td>I can count 20 legs in the paddock. How many ducks and how many sheep are in the paddock? How many solutions can you find? The farmer is taking ducks and sheep to market. Altogether there are 15 heads and 52 legs in the truck. How many ducks and how many sheep are going to market? Download the assessment proforma.</td>
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</tbody>
</table>