# Stage 2 - Addition and Subtraction 2

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<td><strong>A student:</strong></td>
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<tr>
<td>› uses appropriate terminology to describe, and symbols to represent, mathematical ideas MA2-1WM</td>
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<td>› uses mental and written strategies for addition and subtraction involving two-, three-, four and five-digit numbers MA2-5NA</td>
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<tr>
<td><strong>Language</strong></td>
<td>Students should be able to communicate using the following language: plus, add, addition, minus, the difference between, subtract, subtraction, equals, is equal to, empty number line, strategy, digit, estimate, round to, change (noun, in transactions of money). Word problems requiring subtraction usually fall into two types – either 'take away' or 'comparison'. <em>Take away</em> – How many remain after some are removed? eg 'I have 30 apples in a box and give away 12. How many apples do I have left in the box?' <em>Comparison</em> – How many more need to be added to a group? What is the difference between two groups? eg 'I have 18 apples. How many more apples do I need to have 30 apples in total?', 'Mary has 30 apples and I have 12 apples. How many more apples than me does Mary have?' Students need to be able to translate from these different language contexts into a subtraction calculation. The word 'difference' has a specific meaning in a subtraction context. Difficulties could arise for some students with phrasing in relation to subtraction problems, eg ‘10 take away 9’ should give a response different from that for ‘10 was taken away from 9’.</td>
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## Ignition Activities

**Greedy Pig**

1. To play this game you need an ordinary 6-sided die.
2. Each turn of the game consists of one or more rolls of the die. You keep rolling until you decide to stop, or until you roll a 1. You may choose to stop at any time.
3. If you roll a 1, your score for that turn is 0.
4. If you choose to stop rolling before you roll a 1, your score is the sum of all the numbers you rolled on that turn.
5. Each player has 10 turns.
6. The player with the highest score wins.

There are many variations of this game, the most common being a full class version in which the teacher rolls the die, and calls out the numbers. All students play using the same numbers and their score depends on when they elect to ‘save’ their score. If they save their score any further rolls that turn do not count towards their score. If a 1 is rolled all players who have not saved their score get 0 for that turn and the next turn starts. The ones dice can be changed to adding tens, hundreds, thousands or ten thousands.
by writing on blank dice. 1 could be changed to any other number as the key number to avoid rolling.

**First to 100**
One child rolls the dice for the whole class. Other children all have ones/tens chart ruled up. Each child decides whether to put the number in the tens or ones column and find progressive total. First child to make it to 100 wins.

**Race To and From 100**
In pairs, students roll a die and collect that number of popsticks. These are placed on a place value board in the ‘Ones’ column.

**Eg**

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
</table>

The student continues to roll the die, collect popsticks and place them in the Ones column. The total number of popsticks in the ‘Ones’ column is checked and bundled into groups of ten, when ten or more popsticks have been counted. The bundles of ten are then placed in the ‘Tens’ column. When there are ten tens, they are bundled to make one hundred and the game is finished. After the idea of trading is established, students could record the total number of popsticks on the place value board after each roll.

Variation: Students start with 100 popsticks in the ‘Hundreds’ column. As the die is rolled, the number of popsticks is removed from the place value board by decomposing groups of ten. The game is finished when the student reaches zero.

**Variation: students to have a number board up to 10 000.**

**Four Turns To 100**
Organise the students into groups of four. Provide each group of students with a pack of cards in the range 1 to 9. Each player draws a card from the deck and decides if the number they have drawn will represent ones or tens. For example, if a five is drawn it can represent five or fifty. The players take a second draw from the pack, again nominating if the number represents tens or ones and adds the number to their first card. Have the students record their total on an empty number line. Continue the activity until each student has drawn four cards. The player with the highest total not exceeding 100 wins.

**Variations**
Players start at 100 and subtract the numbers, after nominating if the number drawn represents tens or ones. The player closest to zero is the winner.

Players draw two numbers from the pile and make the highest two digit number possible. This becomes their starting number and they continue to play as in the above variation.

*Developing Efficient Numeracy Strategies 2 (DENS 2) pg 80-81*

Cards
Explicit Mathematical Teaching

Students should be encouraged to estimate answers before attempting to solve problems in concrete or symbolic form. There is still a need to emphasise mental computation even though students can now use a formal written method. The following formal method may be used.

**Decomposition**

The following example shows a suitable layout for the decomposition method.

\[
\begin{array}{c}
2456 \\
-1385 \\
\end{array}
\]

\[
\begin{array}{c}
\hline
1071 \\
\end{array}
\]

Word problems requiring subtraction usually fall into two types – either ‘take away’ or ‘comparison’. The comparison type of subtraction involves finding how many more need to be added to a group to make it equivalent to a second group, or finding the difference between two groups. Students need to be able to translate from these different language contexts into a subtraction calculation. The word ‘difference’ has a specific meaning in a subtraction context. Difficulties could arise for some students with use of the passive voice in relation to subtraction problems eg ‘10 takeaway 9’ will give a different response to ‘10 was taken away from 9’.

Revisit strategies for addition and subtraction using two-, three- and four-digit numbers, including:
- the jump strategy eg 23 + 35; 23 + 30 = 53, 53 + 5 = 58
- the split strategy eg 23 + 35; 20 + 30 + 3 + 5 is 58
- the compensation strategy eg 63 + 29; 63 + 30 is 93, subtract 1, to obtain 92
- using patterns to extend number facts eg 5 − 2 = 3, so 500 − 200 is 300
- bridging the decades eg 34 + 17; 34 + 10 is 44, 44 + 7 = 51
- changing the order of addends to form multiples of 10 eg 16 + 8 + 4; add 16 and 4 first

Revisit recording strategies
recording mental strategies eg 159 + 22:
‘I added 20 to 159 to get 179, then I added 2 more to get 181.’
or, on an empty number line

\[
\begin{array}{c}
159 \\
169 \\
179 \\
180 \\
181 \\
\end{array}
\]

Ensure students clearly understand the link between concrete materials and the formal algorithm.
On OHP/interactive whiteboard model how to use the formal algorithm when solving problems.
Demonstrate a variety of problems with/without trading,
use a formal written algorithm to record addition and subtraction calculations involving two-, three-, four- and five-digit numbers, eg

\[
\begin{array}{c}
134 + 2459 + 568 - 1352 + 37049 - \sim 3 \sim \\
235 + 138 + 322 + 168 + 9285 \\
\end{array}
\]
solve problems involving purchases and the calculation of change to the nearest five cents, with and without the use of digital technologies (ACMNA080)

- solve addition and subtraction problems involving money, with and without the use of digital technologies
- use a variety of strategies to solve unfamiliar problems involving money
- reflect on their chosen method of solution for a money problem, considering whether it can be improved
calculate change and round to the nearest five cents
- use estimation to check the reasonableness of solutions to addition and subtraction problems, including those involving money

### Whole Class Teaching Activities

Using the 2008 NAPLAN Year 3 question 15 (Jim is 91 years old. Sam is 8 years old. What is the difference in their ages?), ask “What do I know?” Answer: Jim is 91 years old. Sam is 8 years old. What do I want to know? Answer: What is the difference in their ages?

Think aloud: “I know how old Jim is and I know Sam is 8. I know that difference is another word for take away. I will solve this problem using subtraction”

Model the solution of the problem using concrete materials and/or a drawing and writing the appropriate number sentence on the board. eg

**Step 1:** ‘How many to start with? Jim is 91.’ Put out 91 using tens and ones or draw this.

**Step 2:** ‘How old is Sam?’ Answer: 8. Move 8 ones away or cross out 8 drawn ones. Write 91 – 8 = ‘How many left?’ Answer: 83. Write 91 – 8 = 83

**Step 3:** ‘What is the difference in their ages?’ Answer: 83. Discuss and model other possible solution strategies with the group using counters or drawings.

Guide students to translate the information into a number sentence. 91 take away 8 means I subtract so I use the – sign. I want to know many are left so I use the equal sign =. My number sentence is 91 – 8 = 83

### Mental Strategies

Students are asked to calculate 34 + 17 in their heads. They are then asked to record the strategy they used. This process is repeated for other problems, such as:

- 73 – 25 162 – 69
- 63 + 29 188 – 89

Students discuss which methods are the most efficient.

**Extension:** Students are given increasingly more difficult problems to solve mentally. Students explain and discuss the strategies they use eg for ‘188 – 89 = ?’ A student may say, ‘I took away 88 and that was easy because it left 100 but I had to take away one more, because 88 + 1 = 89, so the answer is 99.’ Students record the mental strategies they use.

Possible questions include:

- is there a better strategy?
- what is the best method to find a solution to this problem?

### Recording on Empty Number Lines

Students are shown the number sentence 157 + 22 and an empty number line. The teacher marks the number 157 on the number line. Possible questions include:
| **What is the next multiple of ten after 157?** |  |
| **How many do you add on to get that number?** |  |
| Students record their answers on the number line. |  |
| Possible questions include: |  |
| **Can you work it out with fewer steps?** |  |
| **Can you visualise the number line in your head and do it?** |  |
| **Can you write the numbers on paper to help you keep track?** |  |

| **Differences on Number Lines** |  |
| In pairs, students draw an empty number line. Student A chooses two three-digit numbers and places them on the number line. Student B uses the number line to work out and record the difference between the two numbers. Students explain the mental strategies they used to find the answer. They reflect on their method, considering whether it can be improved. |  |

| **Using Maths Tracks-Stage 2A, Unit 21-Number** |  |
| One of a series of teaching units to accompany the Rigby/Harcourt series 'Maths Tracks'. Student activities include using mental strategies for addition and subtraction involving two- and three-digit numbers, including using patterns to extend number facts; recording mental strategies. Meets BoS outcomes NS2.2, WMS2.2, WMS2.4. Click on link below. |  |

| **Which Way is Best?** |  |
| Students are asked to solve problems in three different ways: using a mental strategy, a formal written algorithm, and a calculator eg 'Our class has 356 points and another class has 567 points. How many points do we need to catch up?' Students compare the strategies used and discuss the advantages and disadvantages of each method. If students come up with different answers, they are asked to show which answer is correct. |  |
| Variation: Students write their own problems and swap with others. Students could use four-digit or 5-digit numbers. |  |

| **Base 10 Material** |  |
| Students use 2, 3 or 4 dice to generate a two-, three- or four digit number and then represent this number using Base 10 material. Students then generate a second, smaller number by rolling one less die. Students represent this number using Base 10 material, then add the two numbers and show the result using Base 10 material. Students repeat this process, subtracting the second number from the first. Students record their solutions. |  |

| **Number Charts (1-1000)** |  |
| Children learn to add and subtract ones and tens by moving left, right, up and down on charts with numbers up to 1000 eg 45 + 20 etc |  |

| **What went wrong?** |  |
| Students are shown a number of completed subtraction problems with a consistent error eg subtracting the smaller number in a column from the larger number. Students correct the calculations and describe the error that was made. Eg |  |
Students plan how to teach a person who made this mistake a correct method for obtaining solutions.

### Appropriate Calculations

Students are given a calculation such as $160 - 24 = 136$ and are asked to create a number of problems where this calculation would be needed. Students share and discuss response.

### Estimating Addition of Three-Digit Numbers

The teacher briefly displays the numbers 314, 311, 310, 316, 312 on cards, then turns the cards over so that the numbers be seen. Students are asked to estimate the total and give their reasons. The teacher reveals the numbers one at a time so that the students can find the total. The task could be repeated with other three-digit numbers and with four-digit or five-digit numbers.

### Take-away Reversals

In pairs, students choose a three-digit number without repeating any digit and without using zero eg 381. The student reverses the order of the digits to create a second number ie 183. The student subtracts the smaller number from the larger and records this as a number sentence. The answer is used to start another reversal subtraction. Play continues until zero is reached. The process could be repeated for other three-digit numbers. Students discuss their work and any patterns they have observed.

**Extension:** Students repeat using four-digit and 5-digit numbers.

### Estimating Differences

The teacher shows a card with the subtraction of a pair of two-digit numbers eg 78 – 32. Students estimate whether the difference between the numbers is closer to 10, 20, 30, 40 or 50 and give reasons why. The teacher shows other cards eg 51 – 18, 60 – 29, 43 – 25, 33 – 25. Students estimate the differences and discuss their strategies. They are asked to think about rounding numbers on purpose.

For example for 51 – 18, students may round 51 down to 50 and 18 up to 20.

### How Much?

Students are told that a sofa and a desk cost $1116. If the sofa costs $700 more than the desk, how much does the desk cost? Students discuss. Students could pose other similar problems to solve such as ‘What does each item cost if together they cost $1054 and one was $643 more than the other?’

Possible questions include:

- what strategy did you choose to use and why?
- what was the key word/s in understanding the problem?
- how could you check that you have the correct solution?
- could there be more than one solution?

solve problems involving purchases and the calculation of change to the nearest five cents, with and without the use of digital technologies (ACMNA080)
• solve addition and subtraction problems involving money, with and without the use of digital technologies
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Missing Digits
Students are shown a calculation to find the sum of two three-digit numbers, with some of the digits missing.

Eg

\[
\begin{array}{c}
3 \\
2 \\
5
\end{array} \quad + \quad \begin{array}{c}
1 \\
6 \\
0
\end{array}
\]

Students investigate possible solutions for this problem. Students are encouraged to design their own ‘missing digits’ problems. This activity should be repeated using subtraction.

Guided Group and Independent Activities
Please look at Developing Efficient Numeracy Strategies 1 and 2 (blue and orange ones) and Sample Units of Work for more ideas on how to cater for the needs of your different children in your Maths groups

Totalling Up
Hand out containers containing different amounts of money in 5c, 10c, 20c, 50c, $1, $2 coins. Imagine it is money collected from drink sales at a school disco and needs counting. In small groups ask them to count each amount, deciding which way would be best. Deliberately give different assortments of coins to different groups. Ask each group to sort the coins out and total up each amount separately before finding the overall amount, keeping a record as they go. Ask them to think about quick ways of adding it up. Model their methods on the board.
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Think Maths Page 105
Number Cards
Students make number cards from 1 to 9 as shown.

1 2 3 4 5 6 7 8 9

Students use these cards to make two three-digit numbers that add to give the largest total possible and the smallest total possible eg Given 4, 5, 2 and 3, 1, 6:
Largest total possible is 542 + 631 = 1173
Smallest total possible is 245 + 136 = 381
Students arrange the cards to make three three-digit numbers that add up to 999. Students are challenged to find as many solutions as they can.
Variation: students can make numbers up to at least tens of thousands.

Cross-over
In pairs, students each choose a number between 1 and 10 000.
The student with the larger number always subtracts a number from their chosen number. The student with the smaller number always adds a number to their chosen number. The student who is adding must always have a number less than their partner’s answer. The student who is subtracting must always have a number more than their partner’s answer. Play continues until one student is forced to ‘cross over’ their partner’s number. The student who crosses over their partner’s number loses the game.

<table>
<thead>
<tr>
<th>Player A</th>
<th>Player B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start number 135</td>
<td>Start number 899</td>
</tr>
<tr>
<td>135 + 600 = 735</td>
<td>899 – 99 = 800</td>
</tr>
<tr>
<td>735 + 60 = 795</td>
<td>800 – 4 = 796</td>
</tr>
<tr>
<td>795 + 1 = 796</td>
<td>Player B wins</td>
</tr>
</tbody>
</table>

Possible questions include:
- What strategy did you use in solving the addition or subtraction problems?
- Can you find a quicker way to add/subtract?
- Can you explain to a friend what you did?
- How can you show that your answer is correct?
- Does the rule always work?
- Can you use a different method?

Highway Racer
Have the students work in pairs so that each student can explain and verify calculations. Prepare *Highway racer* worksheets for each pair of students. To complete the worksheet, the students take turns to mentally calculate, and record, the number needed to be added or subtracted in order to move to the total written in the next box.
**Variations**

- Have the students create their own “race tracks” for others to solve.
- Have the students verify their partner’s answers using a calculator.
- Have one of the players time his or her partner from “start” to “finish” and then swap roles.
- Have the students “race the clock”. For example, *How far can you move along the track in 60 seconds?*

**Developing Efficient Numeracy Strategies 2 (DENS 2) pg 290-291**
Highway Racer Blackline Master pg 334 (good to copy onto cardboard and laminate)

**Problem Solving**

- Kim’s meal at a restaurant cost half as much as her dad’s meal. Kim and her dad paid $18 altogether for their friends. How much did Kim’s meal cost?
- $3, $12, $6, $9, $18
- solve problems involving purchases and the calculation of change to the nearest five cents, with and without the use of digital technologies (ACMNA080)
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**Engineer’s Dice**

- Provide each group of students with five dice. To play the game a target number is selected by the group. The students then take turns to roll the dice in the following way:
  - Roll all five dice. Choose two of the dice and nominate an operation (+ - x ÷) to carry out with the numbers rolled. Record the result. Discard these two dice.
  - Roll the remaining three dice. Choose one number rolled, complete another operation (+ - x ÷) with the chosen number and the first score. Discard that die.
  - Roll the remaining two dice. Choose one number rolled and complete the same process as the step above using the current total.
  - Roll the last die and complete the same process using the current total.
- After each player has had his or her turn, the students compare their totals to see who is closest to the target score.

**Developing Efficient Numeracy Strategies Stage 2 DENS 2, p. 36**

**Variation:** students may use a calculator or digital technology

**Investigation:**

- How many different ways can you add $5798 + 3565$ in your head? Write number sentences to explain your methods.

**Trading Games**

- The trading games Win 500 or Lose 500 can be adapted for Stage 2 by adding and subtracting two-digit numbers using, recording and evaluating mental strategies. Students are given a pack of playing cards.
cards with the tens and the picture cards removed. The Aces are retained and represent 1 and the Jokers are retained and represent 0. Students flip two cards and assign place values to the numbers turned over. Students play Win 5000/50 000 and Lose 5000/50 000 to add and subtract three-digit and four-digit numbers. Students estimate their answer and then use formal written algorithms. Students could use a calculator to check their answer. Students are encouraged to pose problems, including money problems, using their numbers.

**Computer Learning Objects**

**4 Turns to 100-Stages 1-2**

![Image](4 Turns to 100)

**Addition Wheel –Stages 1-2**

![Image](Addition Wheel)

**Wishball –Stages 2-3**

![Image](Wishball)

**Reference Number L867**

Using Learning Objects To Teach Mathematics’ CD ROM
Or
Count Me In Too website
(Click on link below)
**Planned Assessment**

**Pre Assessment**
Children work out addition/subtraction equations using as many different strategies as they can eg. Jump, split, compensation, bridging, extending number facts, changing order of addends (13 + 15 + 7 = 13 + 7 + 15)

**Count Me In Too : SENA 1, SENA 2**
Select activities to identify where students are on the CMIT framework in number for Addition and subtraction

*What is the question?* Write three difficult questions where 65 is the answer. Draw a circle around what you think is the hardest question for a student to answer

Children solve a variety of two-, three- and four-digit addition and subtraction equations, using as many strategies as they can. Students explain methods.

**Two and three digit addition and subtraction**
Provide the students with a blank piece of paper and ask them to fold the paper into quarters. Write on the board two addition and two subtraction problems, eg 78 + 36, 348 + 189, 95 – 46 and 800 – 241. Ask the students to solve each problem, using a quarter of the paper, recording the strategy they used.

**Work it out in your head**
Download the assessment proforma.

**Strategies for addition and subtraction**
*Explain 3 different ways to solve 257 + ? = 735.*
*Show how you would use an empty number line to solve 63 – 27.*
103 – 47 = 144 *Explain where you think this student made errors.*

**Helping Lee with subtraction**
Download the assessment proforma.

**Adding and subtraction with 2, 3, 4 and 5 digit numbers**
Students fold a piece of paper into 4 sections and write two addition and two subtraction problems provided by the teacher. Students explain how they solved each problem. Download the assessment proforma.
**Sheep and ducks**

I can count 20 legs in the paddock. How many ducks and how many sheep are in the paddock?
How many solutions can you find?

The farmer is taking ducks and sheep to market.
Altogether there are 15 heads and 52 legs in the truck. How many ducks and how many sheep are going to market?

Download the assessment proforma.