## Stage 2 - Multiplication and Division 1

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<tr>
<td>Stage 2</td>
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<td>A student:</td>
<td>uses appropriate terminology to describe, and symbols to represent, mathematical ideas MA2-1WM</td>
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<td>selects and uses appropriate mental or written strategies, or technology, to solve problems MA2-2WM</td>
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<td>checks the accuracy of a statement and explains the reasoning used MA2-3WM</td>
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<td>uses mental and informal written strategies for multiplication and division MA2-6NA</td>
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<td><strong>Language</strong></td>
<td>Students should be able to communicate using the following language: group, row, column, horizontal, vertical, array, multiply, multiplied by, multiplication, multiplication facts, double, shared between, divide, divided by, division, equals, strategy, digit, number chart.</td>
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<td>When beginning to build and read multiplication facts aloud, it is best to use a language pattern of words that relates back to concrete materials such as arrays. As students become more confident with recalling multiplication facts, they may use less language. For example, 'five rows (or groups) of three' becomes 'five threes' with the 'rows of' or 'groups of' implied. This then leads to 'one three is three', 'two threes are six', 'three threes are nine', and so on.</td>
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### Ignition Activities

**Skip counting**
Throughout the focus on this sub strand daily counting by 2's, 3's 5's and 10's is required.

*I'll give you a fact - you give me a fact*
Students stand up at their desks and teacher states a multiplication or division facts of 2, 3, 5 and 10. One by one students give an associated fact - if they get it wrong they sit down. Repeat for next student with another fact. Last left standing wins.

**Beetle Game:** In pairs, students are given ten 'beetles' (or counters) each, two dice (can make up blank dice with different numbers that multiply up to 100) and a hundreds chart game board to share. They roll the two dice and multiply the upper faces. If possible, they place a counter on that position on the hundreds board and change turns (only one counter per square). The winner is the first student with four beetles in a row, column or diagonal who must shout 'Beetle' when they see it (or they don't win!). Support students with a 'times table' to check.

**Multo – 2X, 3X, 5X and 10X**
- Provide each student with a 4X4 grid
- Students write products from 1X1 up to 10X10 in each square
- Roll ten sided dice twice, multiply numbers together
- Students cross off the answer on grids
- First with four in a row win – any direction

Go Maths Stage 2B – Unit 37, Go Maths Stage 2B 37.4
Salute!
This game is played with a pack of cards. One player is the “dealer” who deals a single card to each player. When the dealer deals the cards he/she says “Salute” and the other two players hold the card up to their forehead so that the dealer and the other player can see the card. They aren’t allowed to look at the card dealt to themselves. The dealer multiplies the cards mentally and announces the total. The first player to calculate the number on their own card wins both cards. The winner is the one with the most cards by the end of the deck. The dealer plays the winner and the game continues.

Tables Races
Students make up cards for particular multiplication facts for particular numbers, shuffle them and put them into an envelope eg

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In groups, students are given an envelope of cards. Students race each other to put the cards into order, skip counting aloud. Students state which number has the multiplication facts their cards represent.

Variation: Students write numbers in descending order.

Multiplication Memory
Select a multiple to be practised. Prepare 40 cards, 10 multiplication question cards and 10 division question cards for the selected multiple and 20 appropriate answer cards. Have the students shuffle the cards and place them face down on the floor in four or five rows. The students then take turns to flip over two cards. If a student turns over a question card and the correct answer card then he or she keeps the cards. All players must agree that the cards are a “match”. If the cards do not match then the student flips the cards back over. The player with the most cards wins.

Variation
Have the students create their own set of cards for other multiples.

Explicit Mathematical Teaching
At this Stage, the emphasis in multiplication and division is on students developing mental strategies and using their own (informal) methods for recording their strategies. Comparing their method of solution with those of others, will lead to the identification of efficient mental and written strategies.

One problem may have several acceptable methods of solution.

Use mental strategies to recall multiplication facts for multiples of two, three, five and ten. Relate ‘doubling’ to multiplication facts for multiples of two, eg ‘Double three is six’

Students could extend their recall of number facts beyond the multiplication facts to 10 × 10 by also memorising multiples of numbers such as 11, 12, 15, 20 and 25.

An inverse operation is an operation that reverses the effect of the original operation. Addition and subtraction are inverse operations; multiplication and division are inverse operations.

The use of digital technologies includes the use of calculators.
When beginning to build and read multiplication tables aloud, it is best to use a language pattern of words that relates back to concrete materials such as arrays. As students become more confident with recalling multiplication number facts, they may use less language. For example, ‘seven rows (or groups) of three’ becomes ‘seven threes’ with the ‘rows of’ or ‘groups of’ implied. This then leads to:
- one three is three
- two threes are six
- three threes are nine, and so on.

A student needs to be able to demonstrate an understanding of coordination groups beyond simple recall of number facts. eg. A student able to correctly recall 4 X 7 = 28, may not necessarily be able to determine how many groups of 7 are needed to make up 28.

Developing an understanding of the structure of groups involved in the multiplication and division processes is important. Multiplication tables often appear to be learnt by rote before students develop a concept of multiplication. Students, however, do not use these number facts to solve problems where multiplication is the most efficient strategy unless they have developed an understanding of multiplication as a process.

Multiplication and division concepts are developed when students are able to form equal groups and are able to count using composite groups. An understanding of the Count me In Too Learning Framework in Number provides strategies to enable students to progress from coordinating composite groups using skip counting to using mental computation. The formal algorithm is introduced after students have developed efficient mental strategies for solving tasks involving one digit by a multiple of 10. Arrays are a powerful visual model that can be used to model strategies for multiplication and division and to apply the inverse relationship of multiplication and division to justify answers. Explain why a rectangular array can be read as a division in two ways by forming vertical or horizontal groups, eg 12 ÷ 3 = 4 or 12 ÷ 4 = 3 model and apply the commutative property of multiplication, eg 5 x 8 = 8 x 5
Represent and solve problems involving multiplication using efficient mental and written strategies and appropriate digital technologies (ACMNA057)

- use mental strategies to multiply a one-digit number by a multiple of 10, including:
  - repeated addition, eg $3 \times 20: 20 + 20 + 20 = 60$
  - using place value concepts, eg $3 \times 20: 3 \times 2 \text{ tens} = 6 \text{ tens} = 60$
  - factorising the multiple of 10, eg $3 \times 20: 3 \times 2 \times 10 = 6 \times 10 = 60$
  - apply the inverse relationship of multiplication and division to justify answers, eg $12 \div 3$ is 4 because $4 \times 3 = 12$

- select, use and record a variety of mental strategies, and appropriate digital technologies, to solve simple multiplication problems
- pose multiplication problems and apply appropriate strategies to solve them
- explain how an answer was obtained and compare their own method of solution with the methods of other students
- explain problem-solving strategies using language, actions, materials and drawings
- describe methods used in solving multiplication problems

Whole Class Teaching Activities - some suggestions

Models of the Multiplication Facts

Part A
Students construct models of the multiplication facts using interlocking cubes. They build a staircase eg with 2 blocks in the first step, 4 in the second etc, to represent the multiplication facts for 2. Students use a $10 \times 10$ grid to record their answers.

Part B
Students model the multiplication facts using rectangular arrays and record the associated inverse relationships

eg $3 \times 2 = 6$ and $6 \div 3 = 2$

- 3 rows of 4 is 12
- 4 columns of 3 is 12
- 12 shared into 3 rows is 4
- 12 shared into 4 columns is 3

Variation: Students are given a number (eg 12) and asked to represent all its factors using arrays.

Arrays
On OHP with transparent counters, demonstrate arrays eg showing $3 \times 2$ (3 rows of two counters). Generate the 2x table on the board and ask students if they know what the two times table is about. Ask students to draw a $5 \times 2$ array.
Explain that multiplying is a short cut to repeated addition: ie \(5 \times 2\) means adding 2 five times so we can always work out multiplication by repeated addition but knowing our times table makes it so much faster. Now show the array for \(4 \times 3\). Explain that it means ‘4 lots of 3’, ‘add 3 four times over’. Show each row.

Students may recognise that this is also ‘doubling the double’. If so, discuss why this is the case. That is, doubling means multiplying by two, so to double the double means to multiply by \(2 \times 2\) (ie \(4\)). Show examples such as

\[
4 \times 3 = \text{double (double 3)} \\
= 2 \times (2 \times 3) \\
= 2 \times 6 = 12.
\]

Demonstrate with counters.

Students need to be comfortable with the concept of repeated addition before memorising times tables or it will become a rote learnt skill without understanding. Give them opportunity to draw the arrays or ‘groups of’ to become comfortable with the meaning of multiplication.

Ensure that they can write the number sentence to go with the diagram ie

4 groups of two = \(4 \times 2\)

\[= 8\]

Build an array using the counters from the groups to show the same information.

Allow students in these early days to use repeated addition to find the answer. Discuss what \(4 \times 0\) would look like. Ensure that the zero is always included in a times table - do not assume students know this fact.

Explain why a rectangular array can be read as a division in two ways by forming vertical or horizontal groups, eg \(12 \div 3 = 4\) or \(12 \div 4 = 3\).
• model and apply the commutative property of multiplication, eg \(5 \times 8 = 8 \times 5\)

Students are given an array card. On a separate card they write the number fact for the array. On the back of both cards they write the product. Hang the cards together up around the room.

### Multiplication Facts

Students write the multiplication facts on flash cards from 0 \(\times\) 1 up to 10 \(\times\) 10. In pairs, students test each other to find which facts they can immediately recall and put these into the ‘known’ pile. The others are put into the ‘unknown’ pile. Each day the students concentrate on learning from their ‘unknown’ facts.

Students could repeat this activity with division facts.

Variation: Students play ‘Bingo’ using multiplication and division facts.

### Patterns

Students investigate patterns in the multiplication grid. Students discuss these patterns and record their observations. For example, students compare the multiplication facts for 3 and the multiplication facts for 6. They then investigate the multiplication facts for 9.

Students colour multiples on a hundreds chart and are encouraged to describe the patterns created.

### Doubles

Students work in small groups. A student chooses a small whole number and the next student doubles it. They take turns to keep doubling the number. A student checks the results with a calculator. In the next round they start with a different number.

Possible questions include:
- what did you notice?
- did the pattern help you with your calculations?

### Mental Strategies

Students are asked to write a multiplication fact that they have trouble remembering eg \(8 \times 7\). They are encouraged to try mental strategies to help them recall that fact by using known facts eg ‘I know \(7 \times 7\) is 49 so \(8 \times 7\) must be 7 more than 49 which is 56’ or using the inverse relationship of division: ‘I know 56 ÷ 8 = 7 so \(8 \times 7\) = 56’.

Students are asked to write a division fact they have trouble remembering eg \(36 ÷ 4\). They are encouraged to try mental strategies to help them recall the fact eg using known division facts ‘I know 40 ÷ 4 = 10 so \(36 + 4 = 9\’; using other known facts ‘half of 36 is 18, then if I halve it again I get 9’; using the...
**inverse relationship of multiplication:** \(4 \times 9 = 36 \text{ so } 36 \div 4 = 9\).

### Times Squares
The teacher provides 4 cards using numbers 2, 3, 5 and 10 and places them in a square.

\[
\begin{array}{cc}
4 & 6 \\
3 & 5 \\
\hline
12 & 30 \\
\end{array}
\]

The student multiplies each row and column and records the answers. Students rearrange the cards and record the new multiplication squares.

### Mental Calculations
Students are asked to calculate mentally \(26 \times 4\).
Students discuss the various ways they solved the problem using mental calculation.

\[\text{eg} \quad 3 \times 20: 20 + 20 + 20 = 60\]

- using place value concepts, \(3 \times 20: 3 \times 2 \text{ tens} = 6 \text{ tens} = 60\)
- factorising the multiple of 10, \(3 \times 20: 3 \times 2 \times 10 = 6 \times 10 = 60\)

Mental Calculations include:
- pose multiplication problems and apply appropriate strategies to solve them

### Trading Game with Multiplication and Division
Students play the trading game ‘Race to and from 1000’ with the following variation. Students throw two dice, one numbered 0 to 5 and the other numbered 5 to 10. They multiply the numbers thrown and collect the necessary Base 10 material. The winner is first to 1000.

**Extension:** Students are asked to design their own games involving multiplication and division number facts.

**New From Old**
Students are asked to write a multiplication and a division number fact. Each student uses these facts to build new number facts.

\[\text{eg} \quad \text{Starting with } 12 \div 3 = 4 \quad \text{Starting with } 3 \times 2 = 6\]

\[
\begin{array}{cc}
24 \div 3 = 8 & \quad 6 \times 2 = 12 \\
48 \div 3 = 16 & \quad 12 \times 2 = 24 \\
96 \div 3 = 32 & \quad 24 \times 2 = 48 \\
\end{array}
\]

Possible questions include:
- what strategy did you use?
what other strategies could you use?
what strategy did you use?
did you use the relationship between multiplication and division facts?

**Colour an array**

Provide the students with grid paper and two dice. Tell the students that one die will represent the number of rows and the other die will represent the number of columns. Have each student roll the two dice and then colour in the corresponding number of squares on the grid paper to form an array. The student then cuts and pastes the arrays onto paper and records the number of columns, the number of rows and the total number of squares. Discuss strategies for determining the total. Students may record the information as a number sentence. Allow the students to share and compare their finished work.

Click here to access student worksheet
Extension: Link multiplication and division
Click here to access student worksheet
(pp.100-101 Developing Efficient Numeracy Strategies Stage 2 DENS 2)

**Teddy target**

Draw a large target on the asphalt with chalk. Write the numerals 2, 3, 5 and 10, on the target, so that one numeral is on one segment of the target. Organise the students into teams and provide each team with a set of “teddy bags”. “Teddy bags” are clear plastic bags containing 2, 3, 5 or 10 plastic teddies. Have the students take turns to throw the “teddy bags” onto the target. The team calculates the score by multiplying the number of teddies in the “teddy bag” by the number indicated on the target segment. Have one of the team members record the score and another member check the calculation on a calculator. After each member has had a throw, the team adds the total. The team with the highest total wins.

(pp.200-201 Developing Efficient Numeracy Strategies Stage 2 DENS 2)

**Activities to consolidate understanding of multiplication facts:**

Dice Tables and Four in a Row, Four in a Square (Counting On teaching activities, pp. 164-176)

Explore patterns for multiples and squares on a hundreds chart.
See pp 160-161 Talking About Patterns and Algebra

**Links to Patterning Multiplication and reversibility**
| Multiplication strategies  
         | (p. 93 Talking about Patterns and Algebra) |
|----------|------------------------------------------|
| Array slides  
         | (p. 95 Talking about Patterns and Algebra) |
| Related multiplication and division facts  
         | (p. 97 Talking about Patterns and Algebra) |
| Arrays and multiplication and division facts  
         | (p. 98 Talking about Patterns and Algebra) |
| Square numbers  
         | (p. 106 Talking about Patterns and Algebra) |

Other DENS 2 Activities
- Tasks involving arrays (p98)
- Remainders Count (p278)
- Multiplication memory (p260)
- Multiplication game board (p266)
- Set the rules (p268)
- Una pizza per favora (p256)

Go Maths Investigation
Which 16 different numbers would you want to have on your Multo card? See Appendix 2

Go Maths Investigation 37.4
Investigation Methods to Find Factors
See Appendix 3

Guided Group/Independent Activities-some suggestions

Halves
Students work in small groups. One student chooses a number. The next student halves it. Students take turns as they keep halving. The teacher asks how far they think they can go. A student checks the results with a calculator. Students try starting at a different number when playing the next round.

Tables Races
Students make up cards for particular multiplication facts for particular numbers, shuffle them and put them into an envelope
eg 2,4,6,8,10,12,14,16,18,20 on them
In groups, students are given an envelope of cards. Students race each other to put the cards into order, skip counting aloud. Students state which number has the multiplication facts their cards represent.

**Variation:** Students write numbers in descending order.

### Dominoes
The teacher creates a set of dominoes to be used for practising multiplication facts. Half of the domino has an answer while the other half has two numbers to be multiplied or divided together (to obtain a different answer)
The students try and match the operation with its answer. They play the normal domino rules.

### Double Dice Multi
Provide the students with a baseboard, two dice, one die displaying numerals 1 to 6, the other displaying numerals 1 to 3 and 12 counters.
Instruct the students to take turns to roll the dice and multiply the two numbers rolled. Model to the students how skip counting or repeated addition may be used to determine the answer. Once the answer has been determined, the student covers the corresponding numeral on the baseboard. If the number is already covered, the player misses a turn. Continue until all numerals on the baseboard have been covered.

**Variations**
Modify the dice to have both displaying numerals 1–6 or replace one of the dice with a ten-sided die displaying numerals 1–10. If varying the dice, the base-board will need to be modified.
Use three dice. Have the students roll the dice and choose two of the numbers rolled to multiply.

### Division Number Sentences
Students are asked to devise their own division number sentences with a two-digit number divided by a single-digit number. Students can do this by rolling a die or by choosing the numbers themselves.
Students are asked to model the number sentences with materials and record their number sentences and solutions.
Possible questions include:
- when you were solving a division problem, was there any remainder?
- how did you know?
- how did you record the remainder?

### Previous NAPLAN Questions
Computer Learning Objects

The Array
TaLe Reference Number : L106

Pobble Arrays: Find two factors
TaLe Reference Number : L2058

Pobble Arrays : Make Multiples
TaLe Reference Number : L2056

A group of children share these 21 crayons. Each child gets 3 crayons.

How many children are there in the group?

4 5 6 7

Complete this number sentence to show the total number of wheels in the picture.

[ ] \times 3 = [ ] wheels
Teaching and Learning Exchange (TaLe)
www.tale.edu.au
Type in reference number in search box, click on link when it appears and then click on View
Using Learning Objects To Teach Mathematics' CD ROM
Or
Count Me In Too website
Click on link below)

Ongoing

Multiplication Grid
Students keep a multiplication grid, as shown below. When students are sure they have learnt particular multiplication facts, they fill in that section of the grid. Students are encouraged to recognise that if they know $3 \times 8 = 24$ they also know $8 \times 3 = 24$, and so they can fill in two squares on the grid.

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Planned assessment
Pre Assessment
Give students a multiplication grid to complete in a given time frame
Creating Several Arrays Sample Units of Work pg 93
Students use counters to make an array for a particular number. They create new arrays for this number. Students record their findings eg 20 can be 10 rows of 2 or 4 rows of 5.
Possible questions include:
- how many different arrays can you make?
- how many rows do I have if there are 5 counters in each row? (Adapted from CMIS)

Students are given 3 numbers eg. 15, 20 and 40. They write as many multiplication and division facts about the three numbers as possible.

Pre Assessment
Give students a multiplication grid and tell them they have 3 minutes to fill in as much as they can. Encourage them to start with the ones they know. (Do this frequently, record and compare the results)

Assessment
Give students a multiplication grid with some of the answers given but some of the multipliers missing

Pre Assessment
Write the number 27 as the result of three different number sentences.
Eg 27 = 3 x 9
27 = 2 x 10 + 7
27 = 5 x 5 + 2

Paddocks
Students are given an A4 sheet of paper that has been divided into sections

Area multiplication
Show the cardboard unit square and the “7 x 3” rectangle. How many squares like this would you need to cover the rectangle completely? Provide the student with a copy of the grid and ask:
Can you draw what the squares would look like?
**Work it out in your head**  
Download the assessment proforma.

**Sheep and ducks**  
I can count 20 legs in the paddock. How many ducks and how many sheep are in the paddock?  
How many solutions can you find?  
The farmer is taking ducks and sheep to market.  
Altogether there are 15 heads and 52 legs in the truck. How many ducks and how many sheep are going to market?