## Stage 2 – Patterns and Algebra 2

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<th>Teaching and Learning Activities</th>
<th>Notes/ Future Directions/Evaluation</th>
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<tr>
<td>Stage 2</td>
<td>A student:</td>
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<tr>
<td></td>
<td>‣ uses appropriate terminology to describe, and symbols to represent, mathematical ideas MA2-1WM</td>
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<td>‣ selects and uses appropriate mental or written strategies, or technology, to solve problems MA2-2WM</td>
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<td>‣ checks the accuracy of a statement and explains the reasoning used MA2-3WM</td>
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<td></td>
<td>‣ generalises properties of odd and even numbers, generates number patterns, and completes simple number sentences by calculating missing values MA2-8NA</td>
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### Language
Students should be able to communicate using the following language: pattern, term, missing number, odd, even, number sentence, is the same as, equals.

### Ignition Activities

**Which of these are true?**

Write the following number sentences on the board and ask students to discuss whether or not they are true, and to explain the reasons for their answers. Note the language students use. Model the use of *is the same as* for the equals sign.

- 4 x 3 = 6 x 2
- 4 x 0 = 8 x 0 (Do students generalise about multiplying by zero?)
- 6 x 7 = 7 x 6
- 6 x 2 x 5 = 2 x 5 x 6
- 13 + 11 = 12 + 12
- 18 + 27 = 17 + 28
- 16 + 9 = 9 + 16 (Do students recognise the property or do they do a calculation?)
- 14 + 18 + 20 = 20 + 14 + 18

**What Number Am I Thinking Of?**

Make cards with number sentences on them and cover one number on each card with a flap.
What number is two less than 71?

What number, when divided by four will give you 7?

What number am I thinking of?
If you add five to my number you will get 42.

Later, pose problems such as those above and support students to write number sentences and calculate the missing number. Students should

**Explicit Mathematical Teaching**

Use equivalent number sentences involving addition and subtraction to find unknown quantities (ACMNA083)

- Complete number sentences involving addition and subtraction by calculating missing numbers, e.g., find the missing numbers: $? + 55 = 83$, $? - 15 = 19$
- Use inverse operations to complete number sentences (Problem Solving)
- Justify solutions when completing number sentences (Communicating, Reasoning)
- Find the missing number in a number sentence involving operations of addition or subtraction on both sides of the equals sign, e.g., $8 + ? = 6 + 7$

Investigate and use the properties of even and odd numbers (ACMNA071)

- Investigate and generalise the result of adding, subtracting, and multiplying pairs of even numbers, pairs of odd numbers, or one even and one odd number, e.g., even + odd = odd, odd × odd = odd

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$71 - 2 = \underline{\_\_\_\_\_\_\_\_}$

$37 = \underline{\_\_\_\_\_\_\_\_} - 4$

$\underline{\_\_\_\_\_\_\_\_} \div 4 = 7$

$\underline{\_\_\_\_\_\_\_\_} + 5 = 42$
| Explain why the result of a calculation is even or odd with reference to the properties of the numbers used in the calculation (Communicating, Reasoning) |
| Predict whether the answer to a calculation will be even or odd by using the properties of the numbers in the calculation (Reasoning) |
| Investigate number sequences involving multiples of 3, 4, 6, 7, 8 and 9 (ACMNA074) |
| Generate number patterns using multiples of 3, 4, 6, 7, 8 and 9, eg 3, 6, 9, 12, ... |
| Investigate visual number patterns on a number chart (Problem Solving) |
| Investigate number sequences involving multiples of 3, 4, 6, 7, 8 and 9 (ACMNA074) |
| Explore and describe number patterns resulting from performing multiplication (ACMNA081) |
| Use the word ‘term’ when referring to numbers in a number pattern |
| Describe the position of each term in a given number pattern, eg ‘The first term is 6’ |
| Find a higher term in a number pattern resulting from performing multiplication, given the first few terms, eg determine the next term in the pattern 4, 8, 16, 32, 64, ... |
| Describe how the next term in a number pattern is calculated, eg ‘Each term in the pattern is double the previous term’ (Communicating) |
| Solve word problems by using number sentences involving multiplication or division where there is no remainder (ACMNA082) |
| Complete number sentences involving multiplication and division by calculating missing numbers, eg find the missing numbers: 28 = ? x 7, 40 divided by? = 5 |
| Represent and solve multiplication and division word problems using number sentences, eg ‘I buy six pens and the total cost is $24. What is the cost of each pen?’ can be represented as 6x=? = 24 or 24 divided by 6=? |
| Discuss whether it is more appropriate to represent the problem using or in order to calculate the solution (Communicating, Reasoning) |
| Pose a word problem based on a given number sentence, eg given 4x=? = 28, a problem could be: ‘I have 28 cans of drink and stack them into rows of 4. How many rows will there be?’ |

**Whole Class Teaching Activities**

**Generating Sequences by Counting**

Students generate multiples sequences in different ways. For example, to list the multiples of three, they could silently count two numbers and count out loud every third number as far as 30. After doing this two or three times, ask students to recall the numbers they said out aloud.

Record the numbers on the board as the multiples of three.

Discuss: *Why do you think these numbers are called multiples of three?*

Students work in pairs to continue the sequence and look for patterns. A hundreds chart and transparent counters are used to highlight patterns for multiples of three.

Display the multiples of three sequence. Return to it later and ask questions such as: *What did we call this sequence of numbers? Why?* Cover it up and ask questions such as: *What is the eighth multiple of three? How did you work it out? What are some different ways you could work it out?*

Repeat this sequence for other multiples.

‘Talking About Patterns and Algebra’ pg 71
Generating Sequences by Repeated Addition

Another way to generate multiples sequences is through repeated addition. For example, to generate the sequence of multiples of six, students begin with four and continue adding four.

Begin the sequence as a class activity, writing 4 on the board and asking students to add another four. Write 8 next. Continue this process. Students share their addition strategies and discuss them. The students continue the sequence individually, look for patterns and give the sequence a title. They gather together and discuss the patterns they found. Note the language they use to generalise about patterns.

Write on the board the different titles they have selected and ask students to talk about these. Giving a sequence a title can be another form of generalising it.

*Talking About Patterns and Algebra* pg 73

Generating Whole Number Sequences on a Calculator

Using the constant function on a calculator is another way to generate number sequences. An interesting sequence is the multiples of 37. Begin by writing 37 on the board and asking students to repeatedly add 37. They do the first few mentally, discussing the different mental calculation strategies they use. Generating number sequences provides opportunities for mental addition practice.

After generating the first nine numbers in the sequence, set students to work in pairs to continue the sequence to 999, with the help of the calculator constant function. (To use this function, key in 37 then +, then repeatedly press the = key.) If they list the first nine numbers in a row, then the next nine numbers in a second row and the following nine numbers in another row, they will find that fascinating patterns emerge. Ask students to describe the patterns that they find.

*Talking About Patterns and Algebra* pg 73

Investigating Sequences of Multiples

Students generate a list of multiples and investigate patterns in the sequence. They look for patterns of odd and even numbers, the final digits pattern and patterns in the tens digits.

A useful tool for investigating multiples sequences is the 0-9 wheel on which the sequence of final digits in a list of multiples is mapped.

For example, in the list of multiples of three the final digits sequence is a repeating cycle of 3-6-9-2-5-8-1-4-7-0. Begin...
by drawing a line on the 0–9 wheel from 3 to 6, then continue it to 9 and so on back to 3.

The resulting pattern looks like this.

![Pattern for the seven times table]

**Relating Multiples Sequences**

Students investigate relationships among sequences of multiples, such as relationships among the multiples of two, four and eight, relationships between the multiples of three and six, and relationships among the multiples of three, four and twelve.

Multiples of one and nine produce the same wheel pattern, as do the multiples of two and eight, the multiples of three and seven and the multiples of four and six. Students observe that each pair with the same pattern (e.g. one and nine) adds to ten, and the patterns of the two are drawn in opposite directions.

**Using Maths Tracks-Stage Two-Using Inverse Relationships**

One of a series of teaching units to accompany the Rigby/Harcourt series 'Maths Tracks'. (You don’t the student books to find this resource very useful) Student activities include relating multiplication and division facts; transforming a division calculation into a multiplication problem; completing number sentences involving one operation by calculating missing values; building the multiplication facts to at least 10 x 10 by recognising and describing patterns and applying the commutative property. Meets BoS outcomes PAS2.1, WMS2.2. Includes teacher notes. (Click on link below)


**Investigating Multiplication and Division Facts**
Make some cards with sets of incomplete multiplication or division number sentences such as the following. Students complete the number sentences, discuss their strategies for doing so, and discuss the patterns.

<table>
<thead>
<tr>
<th>3 \times \square = 21</th>
<th>6 \times \square = 42</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 \times \square = 210</td>
<td>6 \times \square = 420</td>
</tr>
<tr>
<td>3 \times \square = 2100</td>
<td>6 \times \square = 4200</td>
</tr>
</tbody>
</table>

**Related Multiplication and Division Facts**

Record some sets of numbers sentences on cards such as the following.

<table>
<thead>
<tr>
<th>4 \times 3 = 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 \times 4 = 12</td>
</tr>
<tr>
<td>12 \div 3 = 4</td>
</tr>
<tr>
<td>12 \div 4 = 3</td>
</tr>
</tbody>
</table>

Ask students: *What can you tell me about these number sentences?* Students make their own cards like these, covering some of the numbers with flaps.

Show cards such as the following and ask students if they can work out the missing numbers and explain the reasons for their answers. Numbers of this magnitude have been chosen to make it more difficult for students to do a quick calculation. They need to look for other strategies, such as relating the facts on the card.

<table>
<thead>
<tr>
<th>3 \times 17 = 51</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 \times \square = 51</td>
</tr>
<tr>
<td>51 \div 3 = \square</td>
</tr>
<tr>
<td>51 \div 17 = \square</td>
</tr>
</tbody>
</table>
Arrays and Multiplication and Division Facts

Display the card from the previous activity, with the missing numbers filled in.

Next, show a card with a 3 x 17 array of dots (without telling students the number of rows and columns) and ask them to work out how many dots there are. Do they refer to the card on display to work out the answer?

Next, show a card with a 6 x 17 array of dots (without telling students the number of rows and columns) and ask them to work out how many dots there are. Do they refer to the card on display and double the 51 when they realise this is a 17 x 6 array?

Set students the task of writing four related multiplication and division facts that the array shows.

Array Slides

Use the overhead projector (OHP) to model the use of the array mat. Arrange the two coloured strips so that they lie on the right and lower boundaries of the 5 x 5 array. Students nominate the number of dots in each row and the number of rows. Students work out the total number of dots in the array. Ask students to make number sentences using multiplication and division. Record the sentences on a retrieval chart with headings:

<table>
<thead>
<tr>
<th>Number of rows</th>
<th>Number of columns</th>
<th>Multiplication facts</th>
<th>Division facts</th>
<th>Interesting facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>5 x 5 = 25</td>
<td>25 ÷ 5 = 5</td>
<td>25 is a square number because the dots form a square</td>
</tr>
</tbody>
</table>

Cover one column of the array so that it represents 4 x 5. Students complete the retrieval chart and explain the reasons for their answers. Move the cover strip to reveal 6 x 5 and repeat the task. Keep moving the cover strip and repeat for two and three columns. Then repeat the
procedure for rows as well. Ask students to explain what they notice about cutting off columns and rows such as the difference between 5 x 5 and 5 x 10.

Provide each pair of students with a 10 x 10 array mat and two cover strips. Students explore the relationships of other arrays to the five by five array and the ten by ten array. Ask them to write statements about the relationships they find. The array chart provides a visual representation of number facts and a link to understanding the concept of area multiplication.

The array chart can also be used to explore the pattern for square numbers.

Investigation:
Tell students that all multiples of 3 have a strange coincidence: when the digits are added the result is also a multiple of 3.

Students use common multiples of 3 and then look at larger multiples of 3 eg 4152 (3 x 1384), 6711 (3 x 2237)

How could we use this knowledge to see which numbers are divisible by 3?
How could we use this knowledge to see which numbers are divisible by 6? (must be even and divisible by 3)

Students may look at the same investigation for multiples of 9.
What other relationships can students find with other numbers?

Show students that different number charts can present different patterns for multiples. Students complete an 11 x 9 chart (attached). Colour in the multiples of 3, 6 and 9. Compare the pattern with the original hundreds chart pattern. Repeat with a 12 x 8 number chart inserting the numbers for the 3x, 6x and 9x tables. Discuss the patterns on the chart.

Predicting odd and even answers
Students will investigate and generalise the result of adding, subtracting and multiplying pairs of even, pairs of odd numbers, and one even or odd number.
Guided Group/Independent Activities

Continuing the patterns
Start a pattern on the floor with concrete materials. Explain the rule used in the pattern and ask them to continue it. After a thorough understanding of the given rule, allow some students to tell you what the rule is and apply it to a new situation.

Apple Picking
- During the holidays, Sue picked apples at the granny smith orchard.
- She was paid:
  - 10c for the first bucket of apples,
  - 20c for the second,
  - 40c for the third and
  - 80c for the fourth.
- If the pattern of payment continued, how much was she paid for the eighth bucket, the tenth bucket, etc?

Calendar
Enlarge a copy of a calendar month. Describe and record horizontal, vertical and diagonal patterns.
- Top-to-bottom diagonal 7+1 or Bottom-to-top 7-1
Highlight a 3x3 grid of nine numbers. Students describe patterns as a group and then cut up the whole calendar to show either the bottom to top diagonal pattern or the top to bottom diagonal chart.

Interactive Technology
Sites2See-Patterns and Algebra
Selected links to a range of interactive and print resources for the Patterns and Algebra strand for K-6 Mathematics. Resources are grouped for Early Stage 1 and Stage 1, Stage 2 and 3 and Teachers.(Click on link below)
http://lrrpublic.clt.det.nsw.edu.au/lrrSecure/Sites/LRRView/10286/10286_00.htm?Signature=(5a3ad8cf-1d93-4806-a1b3-524e9f7830)

Lots of Interactive Activities from this site-Click on Patterns and Algebra up the top
A Maths Dictionary for Kids(Click on link below)

**Hopper: Whole Numbers-Years 3-4**
**TaLe Reference Number : L1084**
The Hopper series of learning objects enables students to investigate patterns in whole numbers and decimals.

![Hopper Image](image)

**Musical Number Patterns:Odds and Evens**
**TaLe Reference Number :L1064**
Make some music by building up rhythms for chimes. Complete a counting rule that matches a pattern on a number line. Select the start number or select a number to count by. For example, start at 1 on a number line; then choose which number to count by (4, 5 or 6) to alternate between odd and even numbers. Add a second number pattern for more chimes. Then play all of the sound patterns together to hear your music. This learning object is one in a series of five objects.

![Musical Number Patterns:Odds and Evens Image](image)

**Musical Number Patterns :Counting Rules**
**TaLe Reference Number: L9834**
Students make counting patterns by following given rules, or use the musical patterns to work out what the counting rules in place are.

![Musical Number Patterns :Counting Rules Image](image)

**Reflection**
Encouraging students to ask questions about patterns builds on understanding and stimulates curiosity and interest in mathematics.

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~ 10 ~
I wonder if and what if type questions encourage students to make conjectures and predictions. Patterns and algebra activities provide opportunities to develop deep knowledge and substantive communication among students. Through discussion, students will develop and use appropriate language to formulate generalisations that are at the heart of algebraic thinking. Patterning activities provide opportunities for exploring number relationships, checking solutions and giving reasons to support conclusions. Students use logical reasoning, proof and justification to apply ideas and form generalisations. Some teacher prompts to develop mathematical thinking include:

*How did you work this out?*
*Why would you do that?*
*What other patterns can you find?*
*Can this be done a different way?*
*Can you predict what the 10th or the 20th number in the pattern might be?*
*Does your pattern always work?*

### Assessment

_Talking About Patterns and Algebra pg 89_

Write the following problem on a large sheet of paper. Imagine a party with four people sharing 1 pizza, 5 cupcakes, 9 sausages and 13 jelly snakes. How much would each person have if they shared everything fairly and there was nothing left over?

Students use their own strategies for sharing the party items. Drawings are a common method of recording.

**Assessment strategy**

The teacher:

- works through the first part of the task with the class
- observes students
- analyses drawings and student work

**Assessment criteria**

The student:

- records a strategy, using drawings, for sharing the party items
- recognises and describes a pattern sequence
- produces a sequence of mixed numbers involving a quarter.

Students give five multiplication or division facts relating to 3, 6 or 9 and one other number to 10.

Provide students with a copy of a calendar. Students describe and record horizontal, vertical and diagonal patterns. Highlight a 3 x 3 grid of nine numbers. Students describe the patterns.

**Classroom observation and discussion notes.**

**Hundred chart patterns**

Provide students with a blank hundreds chart. Students complete the chart and describe patterns.
**Find my pattern**
Repeatedly adding 10 and multiples of 10 Students work in groups to explore: *What happens when you begin with 7 and keep adding ten?* Groups present their findings to the class. (p. 80 *Talking about Patterns and Algebra*).

Investigating computation patterns
(p. 87, p. 102 *Talking about Patterns and Algebra*) Students choose a question to investigate and write an explanation of findings.

**Calendar patterns**
Download the assessment proforma